Mathematics Core Curriculum
Grade 7

In Grade 7, instructional time should focus on four critical areas:

1. Developing understanding of and applying proportional relationships;
2. Developing understanding of operations with rational numbers and working with expressions and linear equations;
3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and
4. Drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of rations and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawing by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of numbers, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other qualities measured in like or different units. For example, if a person walks h mile in each $^1/4$ hour, compute the unit rate as the complex fraction h / $^1/4$ miles per hour, equivalently 2 miles per hour.

2. Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

C. Represent proportional relationships by equations. For example, if total cost "t" is proportional to the number "n" of items purchased at a constant price "p", the relationship between the total cost and the number of items can be expressed as t pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gathering, community celebrations, etc.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

3. Use proportional relationships to solve multistep ratio and percent problems within cultural contexts, including those of

Montana American Indians (e.g., percent of increase and decrease of tribal land). Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

.RP.A.3

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

   a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

   b. Understand p + q as the number located a distance \q\ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

   c. Understand subtraction of rational numbers as adding the additive inverse, p — q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

   d. Apply properties of operations as strategies to add and subtract rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then —(p/q) = (-p)/q = p\(-q). Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in Os or eventually repeats.

3. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers.

Expressions and Equations

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 $^{3}/4$ inches long in the center of a door that is 27 $^{1}/2$ inches wide, you will need to place the bar about 9 inches form each edge; this estimate can be used as a check on the exact computation.

4. Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian cultural contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form PX + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence ofthe operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form PX + q > r or PX + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want you pay to be a least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Geometry

Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians; give an informal derivation of the relationship between the circumference and area of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

6. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

2. Use data, including Montana American Indian demographic data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly samples survey data, predict how many text messages your classmates receive in a day. Gauge how far off the estimate or prediction might be.

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variability's, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Investigate chance processes and develop, use, and evaluate probability modes.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around h indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses.

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

In Grade 8, instructional time should focus on three critical areas:

1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;
2. Grasping the concept of a function and using functions to describe quantitative relationships; and
3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m*A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept oflogical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs oflines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand that statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., f}2). For example, by truncating the decimal expansion of

$\sqrt{}$ 2, show that $\sqrt{}$ 2 is between 1 and 2, the between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 x 3-5 = 3-3

= 1/33 = 1/27.

2. Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where pis a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √ 2 is irrational.

3. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 108 and the population of the world as 7 times 109, and determine that the world population is more than 20 times larger.

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

6. Use similar triangles to explain why the slope mis the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a= a, or a= b results (where a and bare different numbers).

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.

   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 & 6.

   c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

3. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A= s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and 3, 9), which are not on a straight line.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations from a variety of cultural contexts, including those of Montana American Indians:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures from a variety of cultural contexts, including those of Montana American Indians: using coordinates.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem

6. Explain a proof of the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. For example, determine the unknown height of a Plains Indian tipi when given the side length and radius.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data including data from Montana American Indian sources on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

HIGH SCHOOL MATH
ALGEBRA I
9-11

STANDARD 1: NUMBER & QUANTITY

The NUMBER & QUANTITY standard is comprised of the real number system, quantities, the complex number system and vector and matrix quantities. Students will be exposed to yet another extension of **number**, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system— integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

In real world problems, the answers are usually not numbers but **quantities**: numbers with units, which involves measurement. Students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.1NQ.1.RN	**Real Numbers**	**Extend the properties of exponents to rational exponents.** a. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponent. For example, we define 51/3 to be the cube root of 5 because we want (51/3) 3 = 5(1/3)3 to hold, so (51/3) 3 must equal 5. (N-RN.1) b. Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N-RN.2)
HS.M.1NQ.2.RN	**Real Numbers**	**Use properties of rational and irrational numbers.**

a. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (N.RN.3)

HS.M.1NQ.3.NQ	**Quantities**	**Reason quantitatively and use units to solve problems.** a. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (N-Q.1) b. Define appropriate quantities for the purpose of descriptive modeling. (N-Q.2) c. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (N-Q.3)

STANDARD 2: ALGEBRA

The ALGEBRA standard is comprised of expressions, equations and inequalities, and connections to functions and modeling. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.2A.1.SSE	**Seeing Structure**	**Interpret the structure of expressions.** (A-SSE.1)

	In Expressions	a. Interpret expressions that represent a quantity in terms of its context. i. Interpret parts of an expression, such as terms, factors, and coefficients. ii. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. b. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2) 2 – (y2) 2 , thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). (A-SSE.2)
HS.M.2A.2.SSE	**Seeing Structure In Expressions**	**Write expressions in equivalent forms to solve problems.** a. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (A-SEE.3) i. Factor a quadratic expression to reveal the zeros of the function it defines. ii. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. iii. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15t can be rewritten as (1.151/12) 12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
HS.M.2A.3.APR	**Arithmetic with Polynomials and Rational Expressions**	**Perform arithmetic operations on polynomials**. a. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)

HS.M.2A.7.CED	**Creating Equations**	**Create equations that describe numbers or relationships.** a. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1) b. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2) c. Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A-CED.3)
d.		Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. (A-CED.4)
HS.M.2A.8.REI	**Reasoning with Equations and Inequalities**	**Understand solving equations as a process of reasoning and explain the reasoning.** a. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A-REI.1)
HS.M.2A.9.REI	**Reasoning with Equations and Inequalities**	**Solve equations and inequalities in one variable.** a. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A-REI.3) b. Solve quadratic equations in one variable. (A-REI.4)

i. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form.
ii. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

HS.M.2A.10.REI	**Reasoning with Equations and Inequalities**	**Solve systems of equations.** a. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (A-REI.5) b. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (AREI.6) c. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3. (A-REI.7)
HS.M.2A.11.REI	**Reasoning with Equations and Inequalities**	**Represent and solve equations and inequalities graphically.** a. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A-REI.10) b. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or

find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (A-REI.11)

c. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A-REI.12)

STANDARD 3: FUNCTIONS

The FUNCTIONS standard is comprised of interpreting and building functions. It also covers their connections to expressions, equations, modeling and coordinates through linear, quadratic, and exponential models, as well as, trigonometric functions. Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

FUNCTIONS usually have numerical inputs and outputs and are often defined by an algebraic expression. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.3F.1IF	**Interpreting Functions**	**Understand the concept of a function and use function notation.** a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). (F-IF.1) b. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F-IF.2) c. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively

by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. (F-IF.3)

HS.M.3F.2IF	**Interpreting Functions**	**Interpret functions that arise in applications in terms of context.** a. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F-IF.4) b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F-IF.5) c. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (F-IF.6)
HS.M.3F.3IF	**Interpreting Functions**	**Analyze functions using different representations.** a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (F-IF.7) i. Graph linear and quadratic functions and show intercepts, maxima, and minima. ii. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. iii. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. b. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (F-IF.8) i. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

ii. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10 , and classify them as representing exponential growth or decay.

c. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F-IF.9)

HS.M.3F.4BF **Building Functions** **Build a function that models a relationship between two quantities.**

a. Write a function that describes a relationship between two quantities. (F-BF.1)

i. Determine an explicit expression, a recursive process, or steps for calculation from a context.

ii. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

b. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms. (F-BF.2)

HS.M.3F.5BF **Building Functions** **Build new functions form existing functions.**

a. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)

b. Find inverse functions. (F-BF.4)
   i. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.

HS.M.3F.6LE	**Linear, Quadratic & Exponential Models**	**Construct and compare linear, quadratic, and exponential models and solve problems.** a. Distinguish between situations that can be modeled with linear functions and with exponential functions. (F-LE.1) i. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ii. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. iii. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. b. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (F-LE.2) c. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (F-LE.3)

HS.M.3F.7LE	**Linear, Quadratic & Exponential Models**	**Interpret expressions for functions in terms of the situation they model**. a. Interpret the parameters in a linear or exponential function in terms of a context. (F-LE.5)

STANDARD 4: MODELING

MODELING links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.4M.1		**Complete the basic modeling cycle.** a. Identify variables in the situation and selecting those that represent essential features. b. Formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables. c. Analyze and perform operations on these relationships to draw conclusions. d. Interpret the results of the mathematics in terms of the original situation. e. Validate the conclusions by comparing them with the situation and then either improving the model or determining if it is acceptable. f. Report on the conclusions and the reasoning behind the model including the choices, assumptions, and approximations that are present throughout this cycle.

STANDARD 5: GEOMETRY

This standard covers GEOMETRY principles such as congruence, similarity, right triangles and trigonometry, as well as circles. Students also study expressing geometric properties with equations using geometric measurement and dimension, as well as connections to equations through modeling. Students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

No Standards in the Geometry domain.

STANDARD 6: STATISTICS & PROBABILITY

The content covered in this standard include interpreting categorical and quant5itative data, making inferences and justifying conclusions, conditional probability and the rules of probability, as well as using probability to make decisions.

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES

HS.M.6SP.1ID	**Interpreting Categorical & Quantitative Data**	**Summarize, represent, and interpret data on a single count or measurement variable.** a. Represent data with plots on the real number line (dot plots, histograms, and box plots). (S-ID.1) b. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (S-ID.2) c. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S-ID.3)
HS.M.6SP.2ID	**Interpreting Categorical & Quantitative Data**	**Summarize, represent, and interpret data on two categorical and quantitative variables.** a. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S-ID.5) b. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (S-ID.6) i. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. ii. Informally assess the fit of a function by plotting and analyzing residuals. iii. Fit a linear function for a scatter plot that suggests a linear association.
HS.M.6SP.3ID	**Interpreting Categorical & Quantitative Data**	**Interpret linear models.** a. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S-ID.7) b. Compute (using technology) and interpret the correlation coefficient of a linear fit. (S-ID.8)

c. Distinguish between correlation and causation. (S-ID.9)

HS MATH
ALGEBRA II
10-12

STANDARD 1: NUMBER & QUANTITY

The NUMBER & QUANTITY standard is comprised of the real number system, quantities, the complex number system and vector and matrix quantities. Students will be exposed to yet another extension of **number**, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system— integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

In real world problems, the answers are usually not numbers but **quantities**: numbers with units, which involves measurement. Students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.1NQ.4.CN	**Complex Numbers**	**Perform arithmetic operations with complex numbers**. a. Know there is a complex number i such that i 2 = –1, and every complex number has the form a + bi with a and b real. (N-CN.1) b. Use the relation i 2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (N-CN.2)
HS.M.1NQ.6.CN	**Complex Numbers**	**Use complex numbers in polynomial identities and equations.**

a. Solve quadratic equations with real coefficients that have complex solutions. (N-CN.7)

b. Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i). (N-CN.8)

c. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (N-CN.9)

STANDARD 2: ALGEBRA

The ALGEBRA standard is comprised of expressions, equations and inequalities, and connections to functions and modeling. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.2A.1.SSE	**Seeing Structure In Expressions**	**Interpret the structure of expressions.** (A-SSE.1) a. Interpret expressions that represent a quantity in terms of its context. i. Interpret parts of an expression, such as terms, factors, and coefficients.

		ii. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. b. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2) 2 – (y2) 2 , thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). (A-SSE.2)
HS.M.2A.2.SSE	**Seeing Structure In Expressions**	**Write expressions in equivalent forms to solve problems.** a. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. (A-SSE.4)
HS.M.2A.3.APR	**Arithmetic with Polynomials and Rational Expressions**	**Perform arithmetic operations on polynomials**. a. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)
HS.M.2A.4.APR	**Arithmetic with Polynomials and Rational Expressions**	**Understand the relationship between zeros and factors of polynomials.** a. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). (A-APR.2) b. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (A-APR.3)
HS.M.2A.5.APR	**Arithmetic with Polynomials and**	**Use polynomial identities to solve problems.** a. Prove polynomial identities and use them to describe numerical relationships. For

	Rational Expressions	example, the polynomial identity (x2 + y2) 2 = (x2 – y2) 2 + (2xy) 2 can be used to generate Pythagorean triples. (A-APR.4) b. Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. [The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.] (A-APR.5)
HS.M.2A.6.APR	**Arithmetic with Polynomials and Rational Expressions**	**Rewrite rational expressions.** a. Rewrite simple rational expressions in different forms; write a(x) /b(x) in the form q(x) + r(x) /b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. (A-APR.6) b. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. (A-APR.7)
HS.M.2A.7.CED	**Creating Equations**	**Create equations that describe numbers or relationships.** a. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1) b. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2) c. Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints

on combinations of different foods. (A-CED.3)

d. Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. (A-CED.4)

HS.M.2A.8.REI	**Reasoning with Equations and Inequalities**	**Understand solving equations as a process of reasoning and explain the reasoning.** a. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (AREI.2)
HS.M.2A.11.REI	**Reasoning with Equations and Inequalities**	**Represent and solve equations and inequalities graphically.** a. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (A-REI.11)

STANDARD 3: FUNCTIONS

The FUNCTIONS standard is comprised of interpreting and building functions. It also covers their connections to expressions, equations, modeling and coordinates through linear, quadratic, and exponential models, as well as, trigonometric functions. Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

FUNCTIONS usually have numerical inputs and outputs and are often defined by an algebraic expression. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like f(x) = a + bx; or by a recursive rule. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.3F.2IF	**Interpreting Functions**	**Interpret functions that arise in applications in terms of context.** a. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F-IF.4) b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F-IF.5) c. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (F-IF.6)
HS.M.3F.3IF	**Interpreting**	**Analyze functions using different representations.**

Functions

a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (F-IF.7)
   i. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   ii. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   iii. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

b. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (F-IF.8)
   i. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   ii. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10 , and classify them as representing exponential growth or decay.

c. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F-IF.9)

HS.M.3F.4BF **Building Functions** **Build a function that models a relationship between two quantities.**

a. Write a function that describes a relationship between two quantities. (F-BF.1)
   i. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

HS.M.3F.5BF **Building Functions** **Build new functions form existing functions.**

		a. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3) b. Find inverse functions. (F-BF.4) i. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.
HS.M.3F.6LE	**Linear, Quadratic & Exponential Models**	**Construct and compare linear, quadratic, and exponential models and solve problems.** a. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (F-LE.4)
HS.M.3F.8TF	**Trigonometric Functions**	**Extend the domain of trigonometric functions using the unit circle.** a. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (F-TF.1) b. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (F-TF.2)
HS.M.3F.9TF	**Trigonometric Functions**	**Model periodic phenomena with trigonometric functions.** a. Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline. (F-TF.5)

HS.M.3F.10TF	**Trigonometric Functions**	**Prove and apply trigonometric identities.** a. Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to calculate trigonometric ratios. (F-TF.8)

STANDARD 4: MODELING

MODELING links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.4M.1		**Complete the basic modeling cycle.** a. Identify variables in the situation and selecting those that represent essential features. b. Formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables. c. Analyze and perform operations on these relationships to draw conclusions.

d. Interpret the results of the mathematics in terms of the original situation.
e. Validate the conclusions by comparing them with the situation and then either improving the model or determining if it is acceptable.
f. Report on the conclusions and the reasoning behind the model including the choices, assumptions, and approximations that are present throughout this cycle.

STANDARD 5: GEOMETRY

This standard covers GEOMETRY principles such as congruence, similarity, right triangles and trigonometry, as well as circles. Students also study expressing geometric properties with equations using geometric measurement and dimension, as well as connections to equations through modeling. Students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

No standards in the Geometry domain.

STANDARD 6: STATISTICS & PROBABILITY

The content covered in this standard include interpreting categorical and quant5itative data, making inferences and justifying conclusions, conditional probability and the rules of probability, as well as using probability to make decisions.

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.6SP.1ID	**Interpreting Categorical & Quantitative Data**	**Summarize, represent, and interpret data on a single count or measurement variable.** a. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, tables, and Montana American Indian data sources to estimate areas under the normal curve. (S-ID.4)
HS.M.6SP.4IC	**Making Inferences & Justifying Conclusions**	**Understand and evaluate random processes underlying statistical experiments.** a. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (S-IC.1) b. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads

		up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (S-IC.2)
HS.M.6SP.5IC	**Making Inferences & Justifying Conclusions**	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** a. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (S-IC.3) b. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (S-IC.4) c. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (S-IC.5) d. Evaluate reports based on data. (S-IC.6)
HS.M.6SP.9MD	**Probability to Make Decisions**	**Use probability to evaluate outcomes of decisions.** a. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (S-MD.6) c. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (S-MD.7)

HIGH SCHOOL MATH
GEOMETRY
9-11

STANDARD 1: NUMBER & QUANTITY

The NUMBER & QUANTITY standard is comprised of the real number system, quantities, the complex number system and vector and matrix quantities. Students will be exposed to yet another extension of **number**, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system— integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

In real world problems, the answers are usually not numbers but **quantities**: numbers with units, which involves measurement. Students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

No Standards in the Number and Quantity domain.

STANDARD 2: ALGEBRA

The ALGEBRA standard is comprised of expressions, equations and inequalities, and connections to functions and modeling. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

No standards in the Algebra domain.

STANDARD 3: FUNCTIONS

The FUNCTIONS standard is comprised of interpreting and building functions. It also covers their connections to expressions, equations, modeling and coordinates through linear, quadratic, and exponential models, as well as, trigonometric functions. Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

FUNCTIONS usually have numerical inputs and outputs and are often defined by an algebraic expression. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

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No standards in Functions domain.

STANDARD 4: MODELING

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STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.4G.1CO	**Congruence**	**Experiment with transformations in the plane.** a. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and

		distance around a circular arc. (G-CO.1) b. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G-CO.2) c. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (GCO.3) d. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (G-CO.4) e. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G-CO.5)
HS.M.4G.2CO	**Congruence**	**Understand congruence in terms of rigid motions.** a. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G-CO.6) b. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G-CO.7) c. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (G-CO.8)

HS.M.4G.3CO	**Congruence**	**Prove geometric theorems.** a. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G-CO.9) b. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G-CO.10) c. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G-CO.11)
HS.M.4G.4CO	**Congruence**	**Make geometric constructions.** a. Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G-CO.12) b. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (G-CO.13)
HS.M.4G.5SRT	**Similarity Right Triangles & Trigonometry**	**Understand similarity in terms of similarity transformations.** a. Verify experimentally the properties of dilations given by a center and a scale factor: (G-SRT.1)

		i. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. ii. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. b. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G-SRT.2) a. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (G-SRT.3)
HS.M.4G.6SRT	**Similarity Right Triangles & Trigonometry**	**Prove theorems involving similarity.** a. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G-SRT.4) b. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G-SRT.5)
HS.M.4G.7SRT	**Similarity Right Triangles & Trigonometry**	**Define trigonometric ratios and solve problems involving right triangles.** a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G-SRT.6) b. Explain and use the relationship between the sine and cosine of complementary angles. (G-SRT.7)

		c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G-SRT.8)
HS.M.4G.8SRT	**Similarity Right Triangles & Trigonometry**	**Apply trigonometry to general triangles.** a. Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (G-SRT.9) b. Prove the Laws of Sines and Cosines and use them to solve problems. (G-SRT.10) c. Understand and apply the Laws of Sines and Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (G-SRT.11)
HS.M.4G.9C	**Circles**	**Understand and apply theorems about circles.** a. Prove that all circles are similar. (G-C.1) b. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G-C.2) c. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (G-C.3) d. Construct a tangent line from a point outside a given circle to the circle. (G-C.4)
HS.M.4G.10C	**Circles**	**Find arc lengths and area of sectors of circles.** a. Derive using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G-C.5)
HS.M.4G.11GPE	**Geometric**	**Translate between the geometric description and the equation for a conic section.**

	Properties with Equations	a. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G-GPE.1) b. Derive the equation of a parabola given a focus and directrix. (G-GPE.2)
HS.M.4G.12GPE	**Geometric Properties with Equations**	**Use coordinates to prove simple geometric theorems algebraically**. a. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). (G-GPE.4) b. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G-GPE.5) c. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G-GPE.6) d. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (GGPE.7)
HS.M.4G.13GMD	**Geometric Measurement & Dimension**	**Explain volume formulas and use them to solve problems.** a. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G-GMD.1) c. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G-GMD.3)
HS.M.4G.14GMD	**Geometric**	**Visualize relationships between two-dimensional and three-dimensional objects.**

	Measurement & Dimension	a. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G-GMD.4)
HS.M.4G.15MG	**Modeling with Geometry**	**Apply geometric concepts in modeling situations.** a. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone). (G-MG.1) b. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). (G-MG.2) c. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G-MG.3)

STANDARD 6: STATISTICS & PROBABILITY

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Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots.

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Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

HS.M.6SP.6CP	**Conditional Probability & Rules of Prob.**	**Understand independence and conditional probability and use them to interpret data.** a. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (S-CP.1) b. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (S-CP.2) c. Understand the conditional probability of A given B as P(A and B)/P(B) and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (S-CP.3) d. Construct and interpret two-way frequency tables of data, including information from Montana American Indian data sources, when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S-CP.4) e. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S-CP.5)

HS.M.6SP.7CP	**Conditional Probability & Rules of Prob.**	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.** a. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (S-CP.6) b. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. (S-CP.7) c. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. (S-CP.8) d. Use permutations and combinations to compute probabilities of compound events and solve problems. (S-CP.9)
HS.M.6SP.9MD	**Probability to Make Decisions**	**Use probability to evaluate outcomes of decisions.** a. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (S-MD.6) b. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (S-MD.7)

HS MATH
Pre-Calculus & Calculus
11-12

STANDARD 1: NUMBER & QUANTITY

The NUMBER & QUANTITY standard is comprised of the real number system, quantities, the complex number system and vector and matrix quantities. Students will be exposed to yet another extension of **number**, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system— integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

In real world problems, the answers are usually not numbers but **quantities**: numbers with units, which involves measurement. Students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.1NQ.4.CN	**Complex Numbers**	**Perform arithmetic operations with complex numbers**. a. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. (N-CN.3)
HS.M.1NQ.5.CN	**Complex Numbers**	**Represent complex numbers and their operations on the complex plane.** a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. (N-CN.4)

b. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)\ 3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°. (N-CN.5)

c. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. (N-CN.6)

HS.M.1NQ.7.VM	**Vector & Matrix Quantities**	**Represent and model with vector quantities.** a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). (N-VM.1) b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (NVM.2) c. Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors. (N-VM.3)
HS.M.1NQ.8.VM	**Vector & Matrix Quantities**	Perform operations on vectors. (N-VM.4) a. Add and subtract vectors **i.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **ii.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **iii.** Understand vector subtraction v – w as v + (–w) where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate

order, and perform vector subtraction component-wise.

b. Multiply a vector by a scalar. (N-VM.5)
   i. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).
   ii. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v and compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).

HS.M.1NQ.9.VM	**Vector & Matrix Quantities**	**Perform operations on matrices and use matrices in applications.** a. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (N-VM.6) b. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (N-VM.7) c. Add, subtract, and multiply matrices of appropriate dimensions. (N-VM.8) d. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (N-VM.9) e. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (N-VM.10) f. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. (N-VM.11)

g. Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. (N-VM.12)

STANDARD 2: ALGEBRA

The ALGEBRA standard is comprised of expressions, equations and inequalities, and connections to functions and modeling. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.2A.1.SSE	**Seeing Structure In Expressions**	**Interpret the structure of expressions.** (A-SSE.1) a. Interpret expressions that represent a quantity in terms of its context. i. Interpret parts of an expression, such as terms, factors, and coefficients. ii. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. b. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2) 2 – (y2) 2 , thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). (A-SSE.2)
HS.M.2A.2.SSE	**Seeing Structure In Expressions**	**Write expressions in equivalent forms to solve problems.** a. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. (A-SSE.4)
HS.M.2A.3.APR	**Arithmetic with Polynomials and Rational Expressions**	**Perform arithmetic operations on polynomials**. a. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR.1)
HS.M.2A.4.APR	**Arithmetic with Polynomials and Rational Expressions**	**Understand the relationship between zeros and factors of polynomials.** a. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). (A-APR.2)

		b. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (A-APR.3)
HS.M.2A.5.APR	**Arithmetic with Polynomials and Rational Expressions**	**Use polynomial identities to solve problems.** a. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2) 2 = (x2 – y2) 2 + (2xy) 2 can be used to generate Pythagorean triples. (A-APR.4) b. Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. [The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.] (A-APR.5)
HS.M.2A.6.APR	**Arithmetic with Polynomials and Rational Expressions**	**Rewrite rational expressions.** a. Rewrite simple rational expressions in different forms; write a(x) /b(x) in the form q(x) + r(x) /b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. (A-APR.6) b. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. (A-APR.7)
HS.M.2A.7.CED	**Creating Equations**	**Create equations that describe numbers or relationships.** a. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)

		b. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED.2) c. Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A-CED.3) d. Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. (A-CED.4)
HS.M.2A.10.REI	**Reasoning with Equations and Inequalities**	**Solve systems of equations.** a. Represent a system of linear equations as a single matrix equation in a vector variable. (A-REI.8) b. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). (A-REI.9)

<u>STANDARD 3: FUNCTIONS</u>

The FUNCTIONS standard is comprised of interpreting and building functions. It also covers their connections to expressions, equations, modeling and coordinates through linear, quadratic, and exponential models, as well as, trigonometric functions. Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

FUNCTIONS usually have numerical inputs and outputs and are often defined by an algebraic expression. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES

HS.M.3F.3IF	**Interpreting Functions**	**Analyze functions using different representations.**
		a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (F-IF.7)
		  i. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
		b. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (F-IF.8)
		  i. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

		ii. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10 , and classify them as representing exponential growth or decay.
HS.M.3F.4BF	**Building Functions**	**Build a function that models a relationship between two quantities.** a. Write a function that describes a relationship between two quantities. (F-BF.1) i. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
HS.M.3F.5BF	**Building Functions**	**Build new functions form existing functions.** a. Find inverse functions. (F-BF.4) i. Verify by composition that one function is the inverse of another. ii. Read values of an inverse function from a graph or a table, given that the function has an inverse. iii. Produce an invertible function from a non-invertible function by restricting the domain. b. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (F-BF.5)
HS.M.3F.8TF	**Trigonometric Functions**	**Extend the domain of trigonometric functions using the unit circle.** a. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6 and use the unit circle to express the values of sine, cosines, and tangent for x, π + x, and 2π – x in terms of their values for x, where x is any real number. (FTF.3) b. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. (F-TF.4)
HS.M.3F.9TF	**Trigonometric**	**Model periodic phenomena with trigonometric functions.**

	Functions	a. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (F-TF.6) b. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology; and interpret them in terms of the context. (T-TF.7)
HS.M.3F.10TF	**Trigonometric Functions**	**Prove and apply trigonometric identities.** a. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. (F-TF.9)

STANDARD 4: MODELING

MODELING links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

HS.M.4M.1

Complete the basic modeling cycle.

a. Identify variables in the situation and selecting those that represent essential features.
b. Formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables.
c. Analyze and perform operations on these relationships to draw conclusions.
d. Interpret the results of the mathematics in terms of the original situation.
e. Validate the conclusions by comparing them with the situation and then either improving the model or determining if it is acceptable.
f. Report on the conclusions and the reasoning behind the model including the choices, assumptions, and approximations that are present throughout this cycle.

STANDARD 5: GEOMETRY

This standard covers GEOMETRY principles such as congruence, similarity, right triangles and trigonometry, as well as circles. Students also study expressing geometric properties with equations using geometric measurement and dimension, as well as connections to equations through modeling. Students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.4G.11GPE	**Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section.** a. Derive the equations of ellipses and hyperbolas given the foci and directrices. (G-GPE.3)
HS.M.4G.13GMD	**Geometric Measurement & Dimension**	**Explain volume formulas and use them to solve problems.** a. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. (GGMD.2)

STANDARD 6: STATISTICS & PROBABILITY

The content covered in this standard include interpreting categorical and quant5itative data, making inferences and justifying conclusions, conditional probability and the rules of probability, as well as using probability to make decisions.

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.6SP.8MD	**Probability to**	**Calculate expected values and use them to solve problems.**

	Make Decisions	a. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. (S-MD.1) b. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. (S-MD.2) c. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. (S-MD.3) d. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? (S-MD.4)
HS.M.6SP.9MD	**Probability to Make Decisions**	**Use probability to evaluate outcomes of decisions.** a. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. (S-MD.5) i. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. ii. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

HS MATH
TECHNICAL MATH
Grades 9-12

Mathematical Practices

The Standards for Mathematical Practice are essential in the extension of mathematical thinking. Students develop these habits of mind through specific, intentional experiences of writing, reading, talking, and reasoning that connect mathematics to their daily lives and career situations. Even though all of the Standards are important for all quality math courses, the following are highlighted in a technical mathematics course:

- Construct viable arguments and critique the reasoning of others (MP.3)
- Modeling with mathematics (MP.4)
- Attend to precision (MP.6)
- Look for and make use of structure (MP.7)

STANDARD 1: NUMBER & QUANTITY

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In real world problems, the answers are usually not numbers but **quantities**: numbers with units, which involves measurement. Students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.1NQ.3.Q	**Quantities**	**Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

STANDARD 2: ALGEBRA

The ALGEBRA standard is comprised of expressions, equations and inequalities, and connections to functions and modeling. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

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STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.2A.2.SSE	**Seeing Structure In Expressions**	**Write expressions in equivalent forms to solve problems.** a. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (A-SSE.4)
HS.M.2A.7.CED	**Creating Equations**	**Create equations that describe numbers or relationships.** a. Create equations in two or more variables to represent relationships between quantities; graph equations or coordinate axes with labels and scales. (A-CED.2)

		b. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. (A-CED.3)
HS.M.2A.11.REI	**Reasoning with Equations and Inequalities**	**Represent and solve equations and inequalities graphically.** a. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. (A-REI.11)

STANDARD 3: FUNCTIONS

The FUNCTIONS standard is comprised of interpreting and building functions. It also covers their connections to expressions, equations, modeling and coordinates through linear, quadratic, and exponential models, as well as, trigonometric functions. Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

FUNCTIONS usually have numerical inputs and outputs and are often defined by an algebraic expression. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like f(x) = a + bx; or by a recursive rule. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.3F.2IF	**Interpreting**	**Interpret functions that arise in applications in terms of context.**

	Functions	a. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. (F-IF.4) b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (F-IF.5) c. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (F-IF.6)
HS.M.3F.3IF	**Interpreting Functions**	**Analyze functions using different representations.** a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (F-IF.7)
HS.M.3F.4BF	**Building Functions**	**Build a function that models a relationship between two quantities.** a. Write a function that describes a relationship between two quantities. (F-BF.1) i. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.
HS.M.3F.5BF	**Building Functions**	**Build new functions form existing functions.** a. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x - k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph

using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3)

STANDARD 4: MODELING

MODELING links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

STANDARDS	STRAND	GOALS and PERFORMANCE OBJECTIVES
HS.M.4M.1		**Complete the basic modeling cycle.** a. Explain the differences between basic banking accounts interest and finance charges, as well as personal loans. b. Compare and Contrast subsidized and unsubsidized student loans, and different types of mortgages.

c. Utilize patterns with numbers and exponential work, create a model such as planting an apple orchard that will produce better apples, or the growth of mosquitos or other pests.
d. Utilize probability and distributions to explain phenomena such as carnival games, failure rates and quality control, queuing, gerrymandering, sabermetrics, as well as absenteeism and graduation rates.
e. Apply optimization theory in areas such as linear programming, optimal locations, gasoline blending, house flipping, college admittance and circuits (Hamiltonian).
f. Combine math and art to explain tessellations, golden ratio, kites in squares and other ratios.
g. Create a sinusoidal model (daylight hours), quadratic model (CBL readers), linear regression, genetics mapping or barbie bungee.

STANDARD 5: GEOMETRY

This standard covers GEOMETRY principles such as congruence, similarity, right triangles and trigonometry, as well as circles. Students also study expressing geometric properties with equations using geometric measurement and dimension, as well as connections to equations through modeling. Students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

STANDARDS	**STRAND**	**GOALS and PERFORMANCE OBJECTIVES**
HS.M.4G.1CO	**Congruence**	**Experiment with transformations in the plane.** a. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G-CO.2)
HS.M.4G.7SRT	**Similarity**	**Define trigonometric ratios and solve problems involving right triangles.**

	Right Triangles & Trigonometry	a. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G-SRT.8)
HS.M.4G.10C	**Circles**	**Find arc lengths and area of sectors of circles.** a. Derive using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G-C.5)
HS.M.4G.13GMD	**Geometric Measurement & Dimension**	**Explain volume formulas and use them to solve problems.** a. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. (G-GMD.1) c. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G-GMD.3)
HS.M.4G.15MG	**Modeling with Geometry**	**Apply geometric concepts in modeling situations.** a. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone). (G-MG.1)

<u>STANDARD 6: STATISTICS & PROBABILITY</u>

The content covered in this standard include interpreting categorical and quant5itative data, making inferences and justifying conclusions, conditional probability and the rules of probability, as well as using probability to make decisions.

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

STANDARDS STRAND GOALS and PERFORMANCE OBJECTIVES

HS.M.6SP.3ID	**Interpreting Categorical & Quantitative Data**	**Interpret linear models.** a. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S-ID.7) b. Compute (using technology) and interpret the correlation coefficient of a linear fit. (S-ID.8)
HS.M.6SP.9MD	**Probability to Make Decisions**	**Use probability to evaluate outcomes of decisions.** a. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (S-MD.6) b. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (S-MD.7)